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# The Effect of Differences in Gene Frequency on Probability of Paternity 

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#### Abstract

Knowledge of gene frequencies in populations is required for the calculation of probability of paternity. The question remains open as to the degree of accuracy of gene frequency estimates required to give accurate probability of paternity figures. This is of special concern in the HLA system, which has haplotype frequencies known to vary in populations. This paper presents computer simulation data comparing probability of paternity calculations using HLA data from California and North Carolina. Comparisons were made between geographic regions, and between blacks and whites within a geographic region. It was found that when the absolute probability of paternity is high, the average differences induced were small, but at lower probabilities the changes can be large. Differences were most pronounced between black and white populations. Examples of individual cases are given to illustrate the huge differences that can be induced in some cases by changing gene frequency.


KEYWORDS: forensic science, paternity, probability

Calculation of the probability of paternity demands a knowledge of the frequencies of genes in populations. The fact that these frequencies are known with some degree of accuracy allows a calculation of a probability of paternity. Without such data, we could only label men as "possible fathers" with no quantitative measure of the probability of their paternity. Unfortunately, our knowledge of human gene frequencies is less than complete. Although a great many large surveys have been done around the world, we cannot always be sure that the frequencies obtained apply to the population of interest. This is especially true in the United States, where wide differences in ethnic background can exist between adjacent areas. Race and ethnic group can also be problematic factors, since fewer studies have been done on black, Hispanic, and American Indian populations than on white populations. Frequency data for population mixtures and isolated groups can also be difficult to obtain.
Unless the trio (mother, child, and alleged father) in question is drawn from a well-studied population, a laboratory's gene frequency data will probably not be an extremely accurate estimate of the true frequencies in that population. The question then is how accurate must the gene frequencies be to give a reliable probability of paternity? Or, how large a difference in gene frequency is needed to change significantly the probability of paternity? A recent paper by Aiken [1] considers this question a major obstacle for meaningful calculations of probability of paternity.
Previous research [2] has shown that in simple, two allele systems, the probability of pater-
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nity calculations are affected by simulated "errors" in gene frequency only when the frequency of one gene is very low. Hummel and Claussen [3] have explored the same problem in the red blood cell systems. They generated trios by computer and compared paternity probabilities as calculated using Southwest German frequencies (in the form of the Essen-Moller value) to those produced using the appropriate frequencies for various ethnic groups. They report that for most groups, the two Essen-Moller values fall within one "verbal predicate" ${ }^{2}$ of each other. As expected, the degree of coincidence decreases as the distance from Germany, both geographically and ethnically, increases.

In the present paper, an attempt to explore this question is made via a computer simulation for the complex HLA antigenic system. HLA, when used for paternity diagnosis, can have very large effects on the cumulative probability of paternity. This is due to the large number of alleles, all of which are of very low frequency. These traits have induced some worries in the field [5] as to the accuracy of paternity calculations obtained from HLA testing. By examining the effect of gene errors on this case, we hope to obtain an estimate of the extent of change that errors in gene frequency in the HLA system can have on the final product.

## Statistical Methods

To study paternity calculations with a complicated blood group system such as HLA, it is necessary to make some simplifying assumptions. While these assumptions will certainly not invalidate the conclusions made, they may force us to limit our application of the information obtained.

To make the HLA system manageable, consideration was limited to cases in which only one possible father is involved. Thus, theoretical "trios" of mother, child, and alleged father are the subjects. Further, the trios were limited to genotypes in which no homozygotes or "blank" phenotypes appear. Thus, all three subjects have four different and distinct antigens, two at the A locus and two at the B locus. Restricting the subjects to those having four clearly distinguishable antigens means that the true father's genetic contribution to the child is absolutely clear.

A few abbreviations have been used to symbolize the genotypes and haplotypes and their frequencies. The true father of the child must have one haplotype which he passed on to the child. Under the assumptions made, this haplotype is clearly indicated by the child's phenotype. We will call this haplotype, which the true father must possess, the obligatory haplotype, or OH . The other haplotype which the true father possesses, and which is unknown to us, is the complementary haplotype, or CH . To reiterate, the father's genotype is made up of the OH and the CH . We will symbolize the frequencies of these haplotypes in the general population by $f(\mathrm{OH})$ and $f(\mathrm{CH})$. By this symbolism, the frequency of the true father's genotype is the population is $2 \times f(\mathrm{OH}) \times \mathrm{f}(\mathrm{CH})$. We can ignore the homozygous case since our assumptions preclude it.

In our tests for the HLA system, however, we cannot detect the linkage relationships of the antigens. Therefore, for a man accused of paternity, we must calculate the probability that he has the correct haplotypes, given that his phenotype shows the required antigens. We do this by computing the percentage of people in the population with the same four antigens and who have the OH and CH arrangement instead of the opposite arrangement. Thus, for a man who has the antigens A1, A3, B5 and B7, the probability that he has the haplotypes A1, B5 and A3, B7 is:

$$
\frac{2 \times \mathrm{f}(\mathrm{~A} 1 \mathrm{~B} 5) \times \mathrm{f}(\mathrm{~A} 3 \mathrm{~B} 7)}{2 \times \mathrm{f}(\mathrm{~A} 1 \mathrm{~B} 5) \times \mathrm{f}(\mathrm{~A} 3 \mathrm{~B} 7)+2 \times \mathrm{f}(\mathrm{~A} 1 \mathrm{~B} 7) \times \mathrm{f}(\mathrm{~A} 3 \mathrm{~B} 5)}
$$

[^0]or, if we use the OH and CH symbolism:
$$
\frac{2 \times \mathrm{f}(\mathrm{OH}) \times \mathrm{f}(\mathrm{CH})}{2 \times \mathrm{f}(\mathrm{OH}) \times \mathrm{f}(\mathrm{CH})+2 \times \mathrm{f}(\mathrm{~A} 1 \mathrm{~B} 7) \times \mathrm{f}(\mathrm{~A} 3 \mathrm{~B} 5)}
$$

This equation produces a frequency between zero and one. We will call this frequency the probability that a person has the obligatory haplotype given his phenotype, D. We can use this probability to calculate the possibility that a man of this phenotype fathered a child possessing the OH . Since the chance is $50 \%$ that a given haplotype is passed on to a child, the probability is 0.5 (D). To calculate the paternity index, comparing the alleged father's probability of fathering the child to that of a random man, we need the frequency of the OH in the population, or $\mathrm{f}(\mathrm{OH})$. The paternity index is:

$$
\begin{equation*}
\mathrm{PI}=0.5 \times \frac{\mathrm{D}}{\mathrm{f}(\mathrm{OH})} \text { or } \frac{\mathrm{D}}{2 \mathrm{f}(\mathrm{OH})} \tag{3}
\end{equation*}
$$

This is the usual paternity index (PI), a likelihood ratio with no expression of prior probability included. Its derivation and use have been extensively discussed (for example, see Chakraborty [6]).

The true father in a particular case must have given the child the OH . The complementary haplotype the true father possesses is immaterial, and could be any one of the many haplotypes possible in the HLA system. Since the arrangement of antigens into haplotypes is unknown to us, the probability of paternity must depend on the frequencies of both haplotypes, and on the frequencies of the "opposite haplotypes," those composed of the same four antigens in opposite linkage relationship to the OH and CH . Using an inaccurate frequency for any of these four haplotype frequencies will result in an erroneous paternity calculation. If we acknowledge that our gene frequency estimates are always somewhat inaccurate, then we will be concerned with the degree to which this affects the paternity calculations we make.

We can select a haplotype as the obligatory one, the OH , and calculate the paternity indices for all men carrying that haplotype. That is, we study all possible combinations of the OH with other haplotypes (as the $\mathbf{C H}$ ). This can be thought of as calculating the paternity index for all the possible fathers of a child. We can then use the mean of these figures as a gauge of the effects of changing the haplotype frequencies used. If we compare two sets of HLA frequency data and obtain mean paternity indices for various OHs , we can observe the effect of differing gene frequencies on the paternity index.

What can this tell us? By comparing two different sets of gene frequencies, we can tell how important it is to distinguish between them for paternity testing purposes. For example, if gene frequencies from Iowa and Nebraska give very different results, we should be cautious about assuming that Iowa data are sufficient for calculations about Nebraska trios. If black and white data sets give very different results, then extra caution must be taken in determining the racial identity of the trio.

It should be obvious from an examination of Eq 3 that the frequency of the OH and CH will have a large effect on the paternity index. In fact, if $D$ (the probability of the person possessing the OH and CH given the correct phenotype) changes, the paternity index will change by the amount $\Delta D / 2 f(\mathrm{OH})$. The absolute change in the PI is thus highly dependent on the frequency of the obligatory haplotype $f(\mathrm{OH})$. If $f(\mathrm{OH})$ is very small, say 0.0005 , then a change in $D$ of only 0.005 results in a five-point change in the PI. However, if $f(\mathrm{OH})$ is 0.05 , then a change in $D$ of 0.5 is needed to produce the same effect. Thus, if errors in haplotype frequency occur, their effect on the paternity index depends on the frequency of the obligatory haplotype as well as on the size of the error itself. In other words, a small change in a low gene frequency produces a large change in the PI.

The choice of an OH for our simulation will be crucial to the results we get. By choosing an OH which is very different in frequency between the two data sets, we can get a "worst case" look at the change in the paternity indices. By choosing an OH which is identical in frequency in the two data sets, we can gauge how much effect the differences in the other gene frequencies have on the paternity index.

## Procedure

For this research, two sets of HLA gene frequencies were compared. One set is that collected by the Terasaki Laboratory at UCLA, ${ }^{3}$ commonly used as "American" frequencies. The other set is that compiled by Reisner et al [8]. Significant differences between the two data sets were found for several antigens and were reported by Reisner et al [8].

We compared the UCLA data to the North Carolina data by race, and significant differences in haplotype frequencies were noted. A significantly different haplotype frequency is defined as one whose frequency as given by Reisner et al was outside the $95 \%$ confidence interval for the North Carolina data. For this purpose, any haplotypes not found in the North Carolina data were not considered. Fourteen haplotypes were found to fit these criteria in the white population data and twenty in the data on blacks. Additionally, twelve haplotypes which are significantly different in frequency between whites and blacks in the UCLA data were studied. In this comparison, three haplotypes which are very close in frequency in the black and white groups were also selected for study.

For each haplotype chosen as a model OH , all possible phenotypes that included it were generated. For each such phenotype, two possible OH frequencies were used, one from the UCLA data and one from the North Carolina data. All other frequencies in the equation were taken from the UCLA data. The paternity index was computed twice on the basis of the two frequencies available, and a probability of paternity calculated from the paternity index. We chose to express our results as probability of paternity rather than paternity index for these reasons: (1) the probability of paternity is the more generally used figure; and (2) large differences in paternity indices engender very small shifts as the probability approaches $100 \%$. Thus, using a probability rather than a paternity index is a more conservative estimation of differences observed. In the black versus white comparisons, calculations were done using only data for the particular race. That is, calculations were performed independently as one would do for a black trio and for a white trio. The average probability of paternity generated by the various data sets were computed over the 220 possible genotypes generated.

Table 1 gives a sample calculation of the difference in Paternity index between North Carolina and UCLA for a man of phenotype A2, w31; B5, 13, assuming that A2, B5 is passed on to a child. The frequencies of the A2, B5 haplotype in both North Carolina and California are used to calculate the population frequency of persons carrying the correct complement of haplotypes. Since the UCLA data is used as a standard, only the UCLA frequency of the complementary haplotype A31, B13 is used. This example is the case which produces the maximum difference in probability of paternity between the two gene frequency data sets.

Tables 2 and 3 list the haplotypes used, the frequencies observed in North Carolina and California, and the change in the probability of paternity. Data for white and black UCLA populations are given in Table 4.

Tables 5 and 6 give the maximum difference in probability of paternity between North Carolina and UCLA values. This was detected during the calculation of the averages listed in Tables 2 and 3. Table 7 lists representative values for the pairing of A2, B5 with A1, B12 through A36, B13. Table 8 lists the number of probabilities which fell into certain ranges when A2, B5 was paired with all possible haplotypes.

[^1]TABLE 1-Calculation of probability of paternity for alleged father A2, B5/A31, B13.

| Frequency of A2, B5 in NC | 0.0115 |
| :--- | :--- |
| Frequency of A2, B5 in UCLA data | 0.0205 |
| Frequency of A31, B13 in UCLA data | 0.0001 |
| Frequency of A2, B5/A31, B13 in population: |  |
| NC $2 \times(0.0115) \times(0.0001)=0.0000023$ |  |
| UCLA $2 \times(0.0205) \times(0.0001)=0.0000041$ |  |
| But alleged father could also be A2, B13/A31, B5: |  |
| Frequency in UCLA data $=2 \times(0.0061) \times(0.0047)=0.00005734$ |  |
| Probability of A2, B5/A31, B13 $=$ |  |
| NC $0.0000023 /(0.0000023+0.00005734)=0.03856$ |  |
| NCLA $0.0000041 /(0.0000041+0.00005734)=0.06673$ |  |
| Paternity index $=$ |  |
| NC $0.03856 /(2 \times 0.0205)=0.9406$ |  |
| UCLA $0.06673 /(2 \times 0.0205)=1.6276$ |  |
| Probability of paternity $=$ |  |
| NC $0.9406 / 1.9406=48.4 \%$ |  |
| UCLA $1.6276 / 2.6276=61.9 \%$ |  |

TABLE 2-Average change in the probability of paternity between North Carolina and UCLA white populations.

| Obligate <br> Haplotype | Frequency of Obligate Haplotype |  | Resulting Average Probability of Paternity |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | NC | UCLA | NC | UCLA | Difference |
| A3, B15 | 0.0016 | 0.0060 | 96.9 | 98.1 | 3.9 |
| A9, B40 | 0.0016 | 0.0086 | 93.7 | 97.0 | 3.3 |
| A9, B35 | 0.0049 | 0.0142 | 92.5 | 95.1 | 2.6 |
| A2, B35 | 0.0066 | 0.0139 | 91.4 | 93.8 | 2.4 |
| A10, B12 | 0.0016 | 0.0061 | 95.1 | 97.5 | 2.4 |
| A9, B7 | 0.0033 | 0.0143 | 91.3 | 95.2 | 2.1 |
| A2, B15 | 0.0181 | 0.0291 | 91.6 | 93.2 | 1.6 |
| A2, B5 | 0.0115 | 0.0205 | 91.5 | 93.0 | 1.5 |
| A9, B12 | 0.0066 | 0.0147 | 92.2 | 94.5 | 1.3 |
| A3, B40 | 0.0016 | 0.0051 | 96.8 | 98.1 | 1.3 |
| A2, B40 | 0.0197 | 0.0316 | 90.2 | 91.3 | 1.1 |
| A9, B21 | 0.0016 | 0.0056 | 97.2 | 98.2 | 1.0 |
| A10, B27 | 0.0016 | 0.0051 | 97.7 | 98.4 | 0.7 |
| A10, B16 | 0.0066 | 0.0129 | 96.3 | 96.8 | 0.5 |

## Discussion

Examination of the data in Tables 2 and 3 shows that the differences observed between the North Carolina and California haplotype frequencies were quite sufficient to generate changes in the probability of paternity. As expected, the effect was most serious when the haplotype frequencies were small.

In the white population comparisons (Table 2), the probability changes by less than four percentage points, and in half of the cases by less than two percentage points. In the black pop-

TABLE 3-Average change in the probability of paternity between North Carolina and UCLA black populations.

| Obligate <br> Haplotype | Frequency of Obligate Haplotype |  | Resulting Average Probability of Paternity |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | NC | UCLA | NC | UCLA | Difference |
| A9, B35 | 0.0029 | 0.0199 | 81.9 | 92.7 | 10.8 |
| A33, B35 | 0.0029 | 0.0182 | 89.1 | 94.6 | 5.5 |
| A2, B35 | 0.0214 | 0.0359 | 87.5 | 89.4 | 1.9 |
| A3, B35 | 0.0071 | 0.0161 | 93.0 | 94.8 | 1.8 |
| A30, B35 | 0.0114 | 0.0210 | 91.1 | 92.9 | 1.8 |
| A28, B35 | 0.0071 | 0.0155 | 93.4 | 95.0 | 1.6 |
| A1, B21 | 0.0014 | 0.0049 | 97.4 | 98.4 | 1.0 |
| A30, B42 | 0.0100 | 0.0253 | 93.6 | 94.4 | 0.8 |
| A3, B7 | 0.0100 | 0.0185 | 94.2 | 95.0 | 0.8 |
| A28, B7 | 0.0157 | 0.0018 | 99.5 | 98.7 | 0.8 |
| A28, B12 | 0.0114 | 0.0030 | 98.9 | 98.3 | 0.6 |
| A29, B12 | 0.0043 | 0.0100 | 96.7 | 97.3 | 0.6 |
| A30, B5 | 0.0171 | 0.0062 | 98.3 | 97.7 | 0.6 |
| A33, B5 | 0.0129 | 0.0028 | 99.3 | 98.8 | 0.5 |
| A28, B5 | 0.0129 | 0.0041 | 98.9 | 98.5 | 0.4 |
| A10, B17 | 0.0129 | 0.0041 | 98.8 | 98.4 | 0.4 |
| A32, B7 | 0.0100 | 0.0016 | 99.6 | 99.2 | 0.4 |
| A36, B5 | 0.0086 | 0.0016 | 99.6 | 99.4 | 0.2 |
| A31, B5 | 0.0100 | 0.0009 | 99.8 | 99.6 | 0.2 |
| A31, B14 | 0.0071 | 0.0003 | 99.9 | 99.8 | 0.1 |

TABLE 4-Average change in the probability of paternity between black and white UCLA populations.

| Obligate <br> Haplotype | Frequency of Obligate Haplotype |  | Resulting Average Probability of Paternity |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | White | Black | White | Black | Difference |
| A1, B8 | 0.0747 | 0.0127 | 85.4 | 97.0 | 11.6 |
| A3, B7 | 0.0524 | 0.0185 | 88.7 | 95.0 | 6.3 |
| A2, B15 | 0.0291 | 0.0041 | 92.5 | 98.7 | 6.2 |
| A3, B35 | 0.0139 | 0.0359 | 95.4 | 89.3 | 6.1 |
| A30, B42 | 0.0001 | 0.0253 | 99.9 | 94.4 | 5.5 |
| A30, B7 | 0.0007 | 0.0188 | 99.7 | 94.3 | 5.4 |
| A33, B35 | 0.0011 | 0.0182 | 99.7 | 94.6 | 5.1 |
| A30, B17 | 0.0010 | 0.0149 | 99.7 | 95.4 | 4.3 |
| A2, B7 | 0.0332 | 0.0175 | 90.0 | 94.2 | 4.2 |
| A1, B17 | 0.0206 | 0.0042 | 94.9 | 98.4 | 3.5 |
| A36, B35 | 0.0001 | 0.0130 | 99.9 | 96.4 | 3.5 |
| A1, B37 | 0.0021 | 0.0024 | 98.4 | 99.4 | 1.0 |
| A2, B14 | 0.0036 | 0.0036 | 97.8 | 98.6 | 0.8 |
| A2, B17 | 0.0098 | 0.0121 | 96.4 | 95.7 | 0.7 |
| A3, B35 | 0.0221 | 0.0161 | 94.9 | 94.8 | 0.1 |

ulation (Table 3), 14 of 20 haplotypes show changes of 1 percentage point or less. However, several haplotypes show very large changes, up to eleven percentage points.

In the North Carolina versus UCLA average comparisons, no results shifted the probability of paternity from below 95 to above $95 \%$. However, in the comparisons of blacks and whites within the UCLA data set (Table 4), some shifts from above to below $95 \%$ probability were observed. For example, for the haplotype A1, B8, assuming the trio was black produced an aver-

TABLE 5-Obligate and complementary haplotypes producing the maximum change in white population.

| Obligate <br> Haplotype | Frequency of Obligate Haplotype |  | Complementary Haplotype | UCLA <br> Frequency | Resulting Probability of Paternity |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NC | UCLA |  |  | NC | UCLA | Difference |
| A9, B40 | 0.0016 | 0.0086 | A31, B13 | 0.0001 | 42.6 | 79.1 | 36.5 |
| A9, B7 | 0.0033 | 0.0143 | A3, B13 | 0.0005 | 36.3 | 70.1 | 33.8 |
| A10, B12 | 0.0016 | 0.0061 | A29, B22 | 0.0003 | 41.3 | 72.4 | 31.1 |
| A9, B21 | 0.0016 | 0.0056 | A30, B15 | 0.0001 | 49.5 | 77.0 | 27.5 |
| A3, B40 | 0.0016 | 0.0051 | A31, B7 | 0.0030 | 57.7 | 80.8 | 23.1 |
| A9, B35 | 0.0049 | 0.0142 | A11, B21 | 0.0008 | 58.6 | 79.2 | 20.6 |
| A9, B12 | 0.0066 | 0.0147 | A29, 18 | 0.0004 | 37.7 | 57.0 | 19.3 |
| A3, B15 | 0.0016 | 0.0060 | A2, B14 | 0.0036 | 70.5 | 89.3 | 18.8 |
| A2, B35 | 0.0066 | 0.0139 | A33, B40 | 0.0001 | 40.1 | 58.0 | 17.9 |
| A2, B5 | 0.0115 | 0.0205 | A31, B13 | 0.0001 | 48.4 | 61.9 | 13.5 |
| A10, B27 | 0.0016 | 0.0051 | A11, B16 | 0.0011 | 81.0 | 92.5 | 11.5 |
| A2, B40 | 0.0197 | 0.0316 | A31, B13 | 0.0001 | 42.8 | 52.9 | 10.1 |
| A10, B16 | 0.0066 | 0.0129 | A33, B18 | 0.0001 | 83.9 | 90.0 | 6.1 |
| A2, B15 | 0.0181 | 0.0291 | A3, B13 | 0.0005 | 77.3 | 83.0 | 5.7 |

TABLE 6-Obligate and complementary haplotypes producing the maximum change in black population.

| Obligate <br> Haplotype | Frequency of Obligate Haplotype |  | Complementary Haplotype | UCLA <br> Frequency | Resulting Probability of Paternity |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NC | UCLA |  |  | NC | UCLA | Difference |
| A9, B35 | 0.0029 | 0.0199 | A2, B42 | 0.0008 | 25.0 | 68.0 | 43.0 |
| A33, B35 | 0.0029 | 0.0182 | A36, B35 | 0.0003 | 31.1 | 72.3 | 41.2 |
| A30, B5 | 0.0171 | 0.0062 | A2, B42 | 0.0008 | 75.5 | 53.4 | 22.1 |
| A1, B21 | 0.0014 | 0.0049 | A2, B37 | 0.0004 | 68.1 | 87.6 | 19.5 |
| A3, B35 | 0.0071 | 0.0161 | A11, B17 | 0.0004 | 52.2 | 70.3 | 18.1 |
| A28, B35 | 0.0071 | 0.0155 | A36, B14 | 0.0003 | 55.3 | 72.1 | 16.8 |
| A32, B7 | 0.0100 | 0.0016 | A3, B22 | 0.0003 | 95.7 | 79.2 | 16.5 |
| A29, B12 | 0.0043 | 0.0100 | A2, B42 | 0,0008 | 65.6 | 80.8 | 15.2 |
| A30, B35 | 0.0114 | 0.0210 | A11, B42 | 0.0004 | 41.4 | 55.9 | 14.5 |
| A28, B12 | 0.0114 | 0.0035 | A29, B14 | 0.0003 | 91.7 | 78.1 | 13.6 |
| A28, B7 | 0.0157 | 0.0018 | A31, B35 | 0.0007 | 97.7 | 84.7 | 13.0 |
| A2, B35 | 0.0214 | 0.0359 | A33, B12 | 0.0025 | 53.9 | 65.0 | 11.1 |
| A30, B42 | 0.0100 | 0.0253 | A29, B14 | 0.0003 | 75.5 | 86.3 | 10.8 |
| A3, B7 | 0.0100 | 0.0185 | A31, B17 | 0.0005 | 70.0 | 80.1 | 10.1 |
| A10, B17 | 0.0129 | 0.0041 | A3, B22 | 0.0003 | 95.5 | 88.3 | 7.2 |
| A28, B5 | 0.0129 | 0.0041 | A11, B17 | 0.00042 | 95.9 | 89.6 | 6.3 |
| A33, B5 | 0.0129 | 0.0028 | A1, B15 | 0.0003 | 97.9 | 92.5 | 5.4 |
| A36, B5 | 0.0086 | 0.0016 | A32, B35 | 0.0007 | 98.7 | 94.8 | 3.9 |
| A31, B14 | 0.0071 | 0.0003 | A30, B40 | 0.0009 | 99.8 | 96.2 | 3.6 |
| A31, B5 | 0.0100 | 0.0009 | A30, B40 | 0.0009 | 99.5 | 96.0 | 3.5 |

age probability of paternity of $97.0 \%$; the assumption of a white trio produced an average of $85.4 \%$.

Examining the three cases (Table 4) in which the frequency of the OH was similar in both populations (A1, B37; A2, B14; A2, B17), we find, as we would expect, that the degree of change decreases. While differences still exist in the two probabilities, they are all one percentage point or less.

These results produce an interesting comparison to the work of Hummel and Claussen [2]. Using actual trio data for the red blood cell systems, they found very little difference in prob-

TABLE 7-Probability of paternity when obligate haplotype is A2, B5 (white population).

|  |  | Resulting Probability of Paternity |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Complementary <br> Haplotype | UCLA <br> Frequency | NC | UCLA | Difference |  |
| A31, B13 | 0.0001 |  | 48.4696 | 61.9425 | 13.4729 |
| A31, B21 | 0.0003 |  | 64.3142 | 75.22954 | 10.9153 |
| A11, B17 | 0.0008 |  | 65.0576 | 75.7978 | 10.7402 |
| A31, B12 | 0.0023 |  | 65.0868 | 75.8200 | 10.7332 |
| A28, B15 | 0.0008 |  | 65.1975 | 75.9042 | 10.7067 |
| A11, B15 | 0.0020 |  | 65.8658 | 76.4105 | 10.5447 |
| A33, B42 | 0.0001 | 67.0402 | 77.2917 | 10.2515 |  |
| A31, B15 | 0.0011 | 67.3725 | 77.5391 | 10.1666 |  |
| A11, B12 | 0.0051 | 67.6437 | 77.7404 | 10.0967 |  |
| A11, B42 | 0.0001 | 67.8948 | 77.9261 | 10.0313 |  |
| A30, B15 | 0.0001 | 68.6817 | 78.5053 | 09.8236 |  |
| A11, B21 | 0.0008 | 70.5119 | 79.8340 | 09.3221 |  |
| A36, B12 | 0.0002 | 71.1920 | 80.3213 | 09.1293 |  |
| A36, B7 | 0.0001 | 71.6283 | 80.6322 | 09.0039 |  |
| A36, B42 | 0.0001 | 72.5172 | 81.2612 | 08.7440 |  |
| A11, B8 | 0.0010 | 72.6977 | 81.3882 | 08.6905 |  |
| A31, B13 | 0.0001 | 73.3747 | 81.8625 | 08.4878 |  |
| A28, B5 | 0.0038 | 73.9317 | 82.2504 | 08.3187 |  |
| A36, B15 | 0.0001 | 73.9512 | 82.2639 | 08.3127 |  |
| A11, B7 | 0.0037 | 74.8888 | 82.9117 | 08.0237 |  |
| A31, B8 | 0.0006 | 75.4227 | 83.2778 | 07.8551 |  |
| A32, B21 | 0.0003 | 75.4565 | 83.3009 | 07.8444 |  |
| A3, B12 | 0.0065 | 7.9300 | 83.6239 | 07.6939 |  |
| A33, B7 | 0.0002 | 78.2638 | 85.1934 | 06.9296 |  |
| A28, B17 | 0.0007 | 78.4201 | 85.2972 | 06.8771 |  |
| A11, B13 | 0.0009 | 79.1571 | 85.7846 | 0.6275 |  |
| A1, B42 | 0.0001 | 79.7289 | 86.1602 | 06.4313 |  |

TABLE 8-Breakdown for probability of paternity when A2, B5 is obligatory haplotype (white population).

| Probability <br> Range | Number of Calculations <br> for NC Data | Number of Calculations <br> for UCLA Data |
| :--- | :---: | :---: |
| $95.0+$ | 27 | 51 |
| $90.0-94.9$ | 75 | 118 |
| $80.0-89.9$ | 91 | 39 |
| $70.0-79.9$ | 16 | 11 |
| Less than 69.9 | 11 | 1 |
| Total | 220 | 220 |

ability of paternity when different population gene frequencies were used. Our results for the average probability of paternity agree with their observations, but we produced some very large differences in individual cases. This could be explained by the fact that we have chosen to work with the HLA system, a system most likley to show differences because of the large number of genes of very low frequency. Also, we chose to study cases in which the difference between populations was known to be large.

Perhaps the most important results of this research can be seen in Tables 5 and 6, listing the maximum difference in the probability of paternity which was observed in the simulation for each haplotype. Even though for the obligate haplotypes we chose the average probability of paternity is fairly high and the differences induced small, in individual cases the probability of
paternity can be quite low and the difference very great. For example, although the average probability of paternity for the A2, B5 haplotype in whites is $91.5 \%$ using the North Carolina frequency and $93.0 \%$ using the California frequency, the respective probabilities are only 48.4 and $61.9 \%$ when A2, B5 is paired with A31, B13. While examination of Tables 7 and 8 makes clear that this is an isolated case, it is of concern to workers in the field of paternity testing that such large differences can occur.

## Conclusion

We can draw several conclusions from the simulations performed. The most important haplotype frequency to consider when frequency data are of questionable accuracy is the one required to be passed to the child from the father. If the frequency of the obligatory haplotype is changed significantly, the probability of paternity must change. When the average probability of paternity is high, a change in the obligatory haplotype frequency does not significantly change that probability but at lower levels of the average probability of paternity, especially below the $90 \%$ level, a change in the frequency of the obligatory haplotype can change the probability of paternity by up to twelve percentage points in our study. Thus if the obligatory haplotype in a particular case is known to vary significantly in different populations, the probability of paternity calculation may be less dependable. Even if the obligatory haplotype frequency is stable in different populations, small variations in the probability of paternity may occur due to the different frequencies in the "background" population data. Given the small size of the variations observed in this study, this is not likely to be of major concern.

It must be remembered that these conclusions are based on averages over many data points. Examination of the maximum differences observed shows that tremendous changes in the probability of paternity can be brought about by gene frequency errors. It is always advisable if there is reason to suspect that other gene frequencies may be applicable to calculate the paternity indices, giving all probable frequencies. For example, if questions exist on the race of the father, calculations should be made assuming both white and black gene frequencies.

It is also important to note the simplifying assumptions which were made for this work. These conclusions, while fairly reassuring, may not hold in other cases. While including blank antigens and homozygotes does not present any inherent reasons for altering the results we have obtained, it does present the possibility of further complication such as inbreeding.

It can be concluded that erroneous estimates of gene frequency can have severe effects on the calculation of a probability of paternity when using the HLA system. While this research has examined what might be thought of as a worst case situation, the UCLA population data are occasionally used as "representative" American values, and we have shown that North Carolina populations are quite different. The results also advise us to use extreme caution in the assignment of race/ethnic group to subjects being tested, since the difference between black and white probabilities can be very large for certain haplotypes.

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[^0]:    ${ }^{2}$ "Verbal predicates" are categorical guidelines designed to be used for paternity decisions. Originated by Hummel et al [4], they are assigned on the basis of the probability of paternity calculation, but are not based on statistical principles.

[^1]:    ${ }^{3}$ We have used an early edition of data dated 2 Sept. 1980. A modified version of the same data is included in the paper by Dykes [7].

